Technical Comments

Comments on "Elliptic Elements in Terms of Small Increments of Position and Velocity Components" and Generalization of the Solutions for an Arbitrary Initial Point

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THE valuable solutions of de Vries[1] to the "rendezvous" equations for a reference elliptic orbit of small eccentricity unfortunately contain a few algebraic errors. The following terms should be changed to read

$$e_0\,(\tfrac{3}{2}B - 2B\,\cos v_0 - 6C\,v_0\,\cos v_0)$$

instead of

$$e_o(+\,2B\,\cos v_0 - 6\,v_0\,\sin v_0)$$

in the y equation, and

$$e_o(-\tfrac{1}{3}E\,\cos v_0 - \tfrac{1}{6}E\,\cos 2\,v_0)$$

instead of

$$e_o(-\tfrac{1}{6}E\,\cos v_0 - \tfrac{1}{3}E\,\cos 2v_0)$$

in the z equation.

deVries' solutions are restricted to an initial point located at perifocus of the reference elliptic orbit. The following solutions for x, y, and z are generalized to an arbitrary initial point. The symbols e and v correspond to e_0 and v_0 in Ref. 1 and $v(0)$ is the true anomaly of the initial point ($t = 0$) on the reference elliptic orbit.

Solutions to the small eccentricity, elliptic "rendezvous" equations of relative motion for an arbitrary initial point follow:

$$\begin{aligned} x = {} & 2C + A\,\cos v + B\,\sin v + e\{-\tfrac{3}{2}A + \tfrac{1}{2}A\,\cos 2v + \\ & \tfrac{1}{2}B\,\sin 2v - 3C\,v\,\sin v + \tfrac{1}{2}A[1 + \cos 2v(0)]\,\cos[v - \\ & v(0)] + 2A\,\sin v(0)\,\sin v - B\,\cos v(0)\,\sin v + \tfrac{1}{2}B[1 - \\ & \cos 2v(0)]\,\sin[v - v(0)] - \tfrac{3}{2}C\,\cos v + \tfrac{3}{2}C\,\cos[v - \\ & 2v(0)]\,3C\,v(0)\,\sin v\} \end{aligned}$$

$$\begin{aligned} y = {} & D - 3C\,v - 2A\,\sin v + 2B\,\cos v + e\{3A[v - \\ & v(0)] - \tfrac{1}{2}A\,\sin 2v + \tfrac{1}{2}B\,\cos 2v + 9C\,\sin v - 6Cv\,\cos v - \\ & A[1 + \cos 2v(0)]\,\sin[v - v(0)] + 4A\,\sin v(0)\,\cos v - \\ & 2B\,\cos v(0)\,\cos v + B[1 - \cos 2v(0)]\,\cos[v - v(0)] - \\ & 3C\,\sin[v - 2v(0)] + 6C\,v(0)\,\cos v - \tfrac{3}{2}A\,\sin 2v(0) + \\ & \tfrac{3}{2}B\,\cos 2v(0) - 12C\,\sin v(0)\} \end{aligned}$$

$$\begin{aligned} z = {} & E\,\cos v + F\,\sin v + e\left\{\frac{E}{2} - \frac{E}{6}\cos 2v - \frac{F}{6}\sin 2v - \right. \\ & \frac{E}{3}\cos^2 v(0)\,\cos[v - v(0)] - \frac{2}{3}E\,\sin v(0)\,\sin v + \frac{F}{3}\cos v(0)\,\times \\ & \left.\sin v - \frac{F}{6}\,[1 - \cos 2v(0)]\,\sin[v - v(0)]\right\} \end{aligned}$$

where

$$A = -\,[3x(0) + 2y'(0)]\,\cos v(0) - x'(0)\,\sin v(0)$$

$$B = -\,[3x(0) + 2y'(0)]\,\sin v(0) + x'(0)\,\cos v(0)$$

$$C = 2x(0) + y'(0)$$

$$D = y(0) - 2x'(0) + 3v(0)\,[2x(0) + y'(0)]$$

$$E = z(0)\,\cos v(0) - z'(0)\,\sin v(0)$$

$$F = z(0)\,\sin v(0) + z'(0)\,\cos v(0)$$

Reference

[1] deVries, J. Pieter, "Elliptic elements in terms of small increments of position and velocity components," AIAA J. 1, 2626–2629 (1963).

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A Computation of One-Dimensional Combustion of Methane

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SEVERAL problems involving the analysis of chemical kinetics have been considered recently. Particularly interesting are the problems in combustion. Several models have been examined by different authors,[1–3] and it is easy to foresee that more and more complex and sophisticated chemical phenomena will attract the researcher's interest in the future.

In gasdynamics, one assumes some basic information from chemistry, viz., the number of chemical species to be considered, the number and nature of the reactions, and the numerical values of forward and backward reaction rates. The problem is that of determining the chemical composition (i.e., the mass fraction of each specie) as a function of time, by integrating the differential equations that express the rate of production of each specie.

As pointed out in Ref. 4, the integration techniques for nonlinear, ordinary differential equations, based on Runge-Kutta or similar methods, are, in this case, handicapped by a stability criterion. The more complex the chemical phenomenon, the longer becomes the time needed on a computing machine to perform the numerical analysis.

The technique presented by Moretti seemed to circumvent that difficulty and provide an increase in the allowable stepsize of several orders of magnitude.

Such a technique was based upon a manipulation of the nonlinear differential equations which reduces them to the linear form

$$y_i' = dy_i/dt = \Sigma a_{ij}y_j + b_i \qquad (1)$$

where y_i is the concentration in moles per cubic centimeter of the ith species, t is time, and a_{ij} and b_i are constant during a time interval from t_n to t_{n+1}. The numerical solution of these equations requires the evaluation of the complex eigenvalues and eigenvectors of an nth order real matrix.

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